NONLINEAR DYNAMICS ON THE CHEAP IN THE JUNIOR LABORATORY

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Background

- This past spring (2015) three of us (RK, RT, JC) team-taught a Junior level laboratory in which physics majors are required, over two semesters, to complete experiments in 8 out of 10 main topic areas of physics.

- As of the beginning of the spring 2015 semester, a satisfactory Mechanics/Nonlinear Dynamics experiment supplied with instructions had not been identified.

- Towards the end of the semester two Metropolitan State University of Denver students (NH, JZ) expressed an interest in investigating the nonlinear dynamics of Duffing’s oscillator.
Why Duffing’s Oscillator?

• Rich History:
  \[ \ddot{\theta} + \delta \dot{\theta} + \beta \theta + \alpha \theta^3 = \gamma \cos \omega t. \]

• Wide Ranging Applications:
  – Science, engineering, economics, even music
  – Defies description by “standard” methods (e.g. – superposition)

• Can be applied to a range of experiments

• Exhibits Nonlinear Dynamics/Chaos
Novel Design
Other Designs
(Metro Student – Dane Beck)
DATA COLLECTION

- Pasco Rotary Sensor
- Data Studio Software (1000 Hz – sample rate)
- Sinusoidal Function Generator to Drive the Helmholtz Coils (1 – 2 Hz)
- Support Springs to Vertically Balance Permanent Magnets on Flexible Rod (Inverted)
- 20 min. runs
- Data taken at 0.025 sec. intervals
- Velocity: time-centered average with 0.05s time step
Analysis

• 1.2 million data points per typical run
  – Open source software: SciLab (developed in Europe)
  – MatLab like
  – (http://www.scilab.org/)

• SciLab for simulation/analysis:
  – Scilab is free and open source software for numerical computation providing a powerful computing environment for engineering and scientific applications.
  – Scilab is released as open source under the CeCILL license (GPL compatible), and is available for download free of charge.
Experimental Time Series – 1.2 Hz
Theoretical Time Series
Experimental Phase Plot – 1.2 Hz
Theoretical Phase Plot
Experimental Poincare Section – 1.2 Hz Strange Attractor
Theoretical Poincare Section
Strange Attractor
Poincare Theoretical

Poincare Experimental – 1.2 Hz (0.000, 0.083, 0.167, 0.250, 0.333, 0.417, 0.500, 0.583, 0.667, 0.750 [t/T])
Other Behavior
(Time Series: 1.4 Hz)
Other Behavior

Phase Plot – 1.4 Hz
- Not Chaotic

Poincare Section – 1.4 Hz
- Period 6
Discussion

• Novel Duffing’s oscillator apparatus
• Simple mechanism shows chaos
• Rich experiment with reasonable results
• Apparatus is CHEAP and easy to build from parts in most undergraduate laboratories
• Can be coupled with simulation
• And – did I mention – it was really cheap!
References

- http://www.scilab.org/scilab/about