NONLINEAR DYNAMICS ON THE CHEAP IN THE JUNIOR LABORATORY

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Background

- This past spring (2015) three of us (RK, RT, JC) team-taught a Junior level laboratory in which physics majors are required, over two semesters, to complete experiments in 8 out of 10 main topic areas of physics.

- As of the beginning of the spring 2015 semester, a satisfactory Mechanics/Nonlinear Dynamics experiment supplied with instructions had not been identified.

- Towards the end of the semester two Metropolitan State University of Denver students (NH, JZ) expressed an interest in investigating the nonlinear dynamics of Duffing's oscillator.

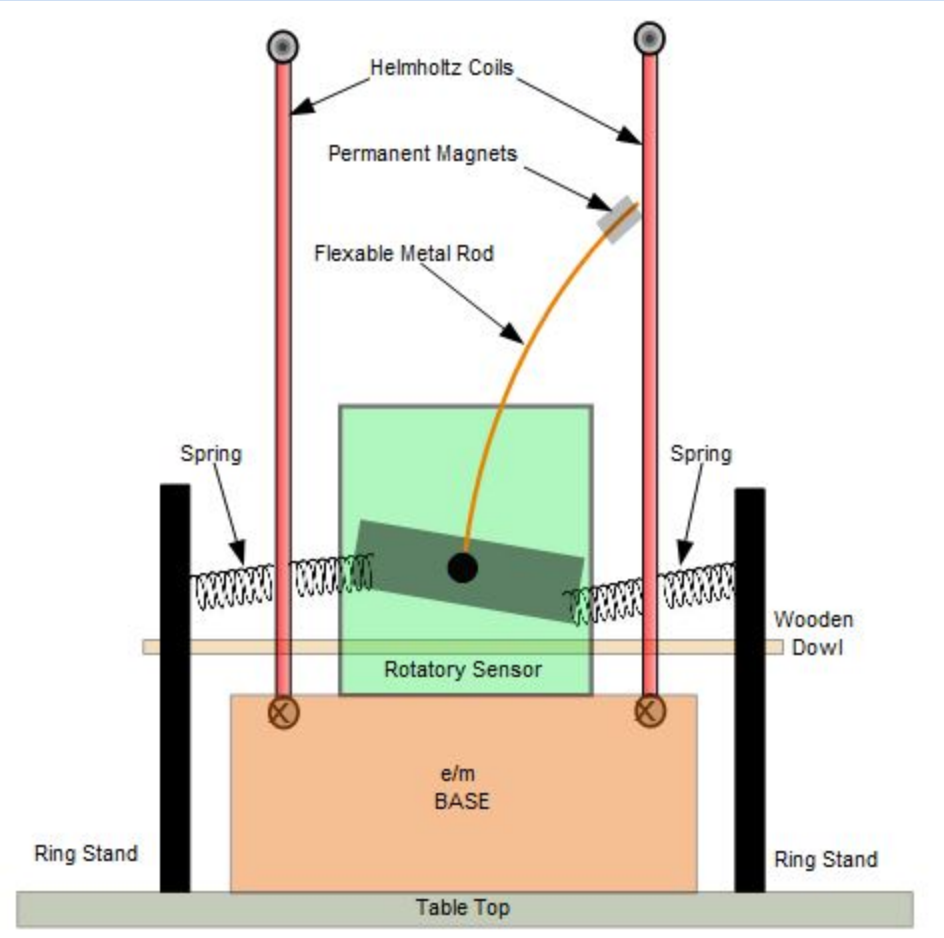
Why Duffing's Oscillator?

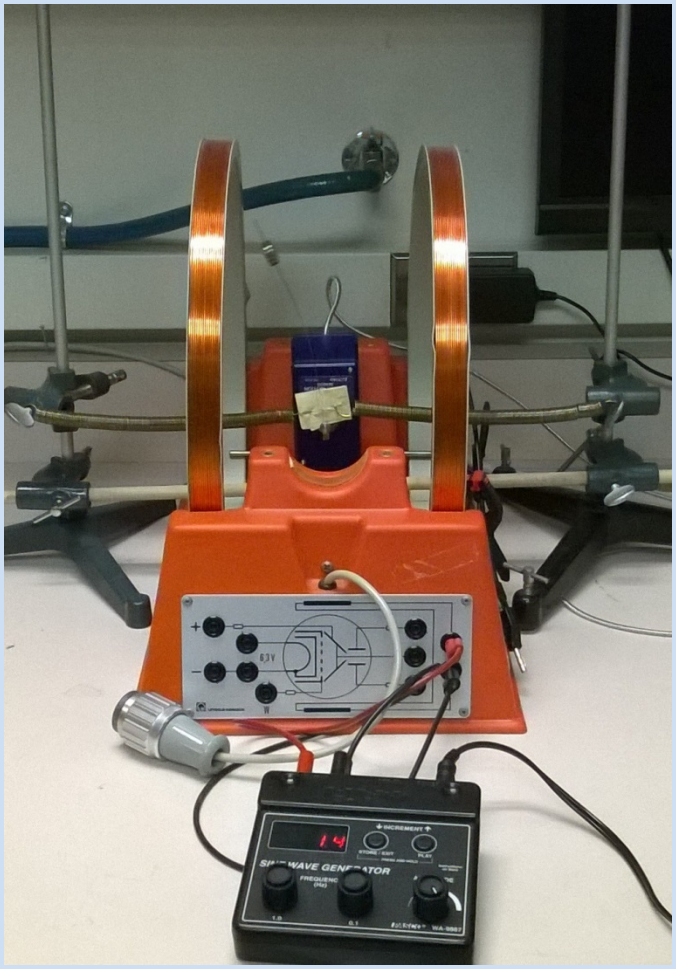
- Rich History:

$$\ddot{\theta} + \delta\dot{\theta} + \beta\theta + \alpha\theta^3 = \gamma \cos\omega t.$$

- Wide Ranging Applications:
  - Science, engineering, economics, even music
  - Defies description by "standard" methods (e.g. – superposition)
- Can be applied to a range of experiments
- Exhibits Nonlinear Dynamics/Chaos

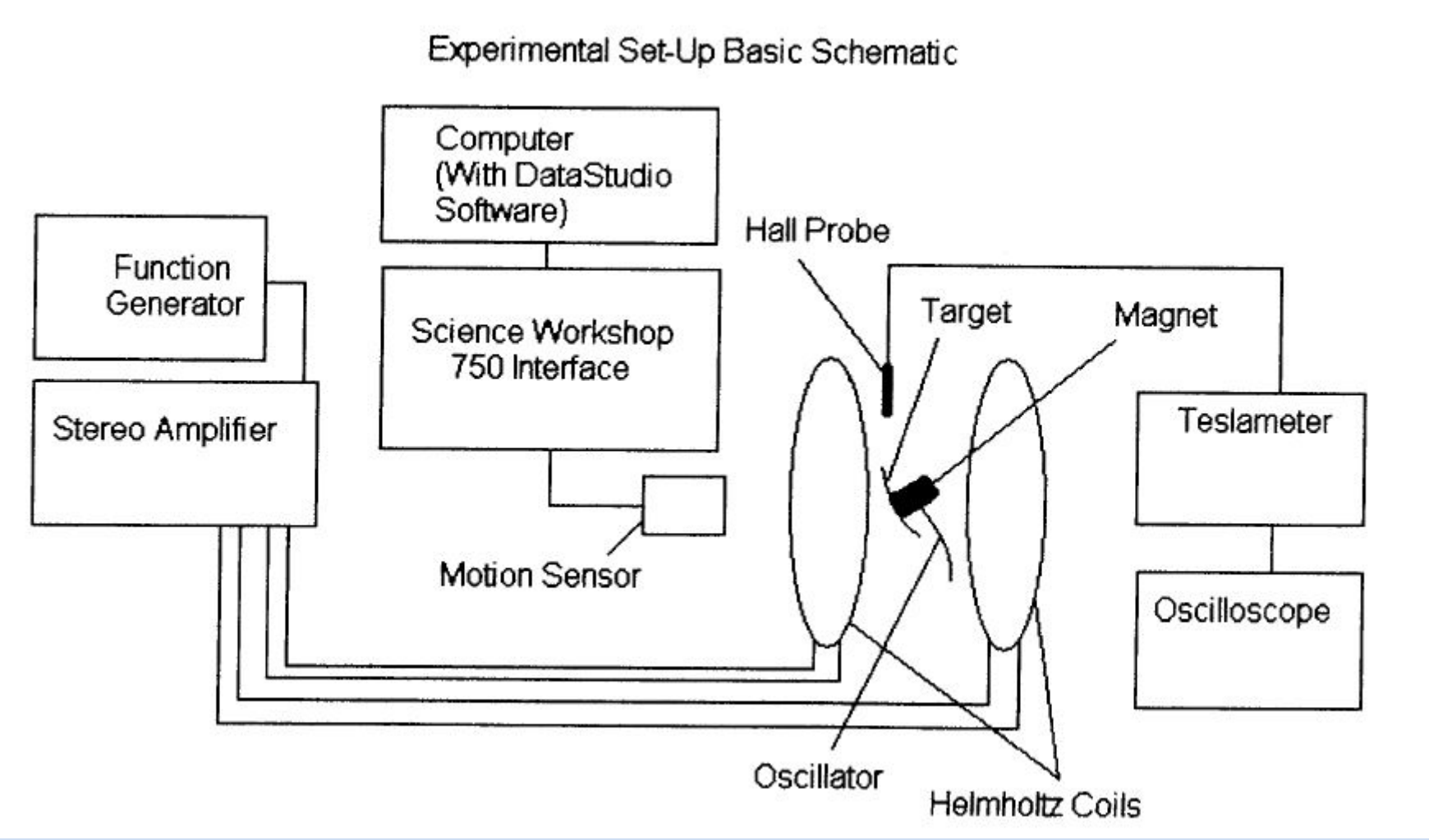
Novel Design



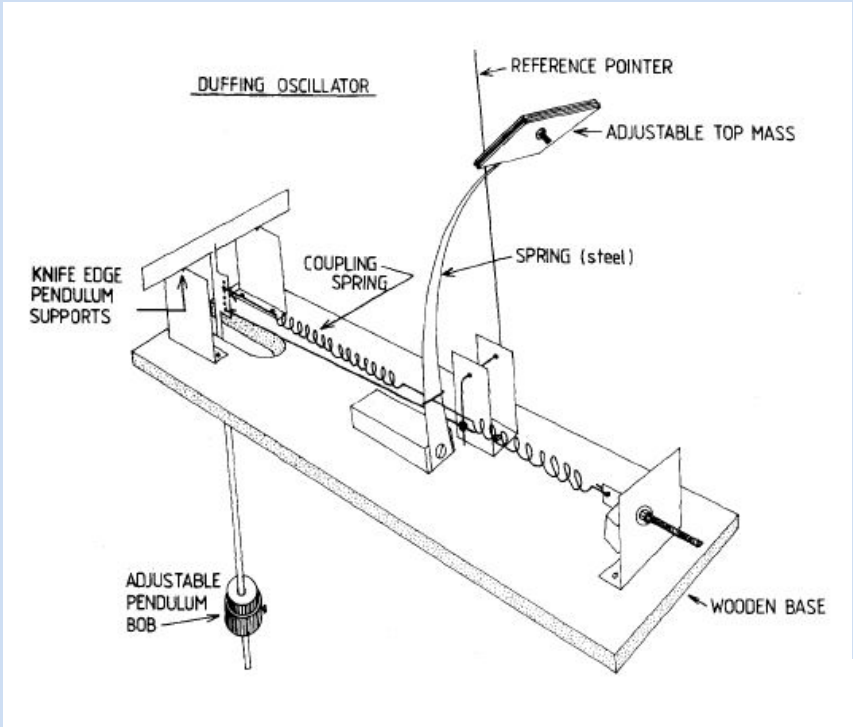


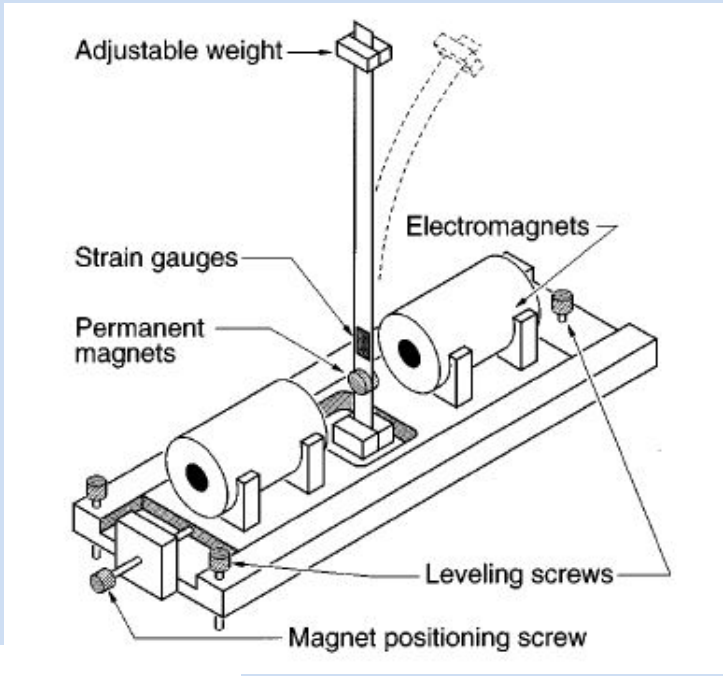
Other Designs

(Metro Student – Dane Beck)



From the Literature





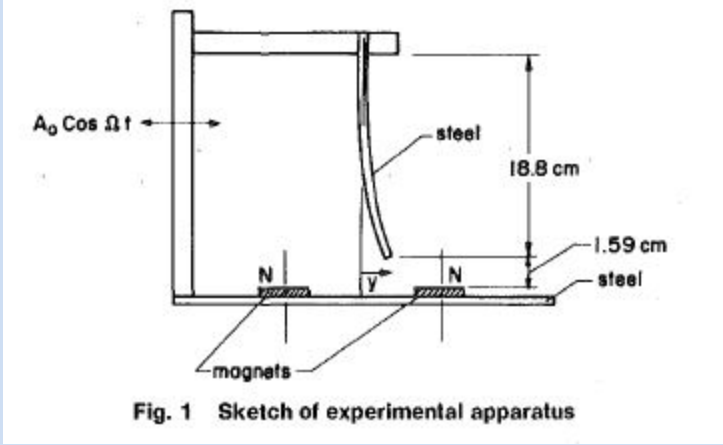


Fig. 1 Sketch of experimental apparatus

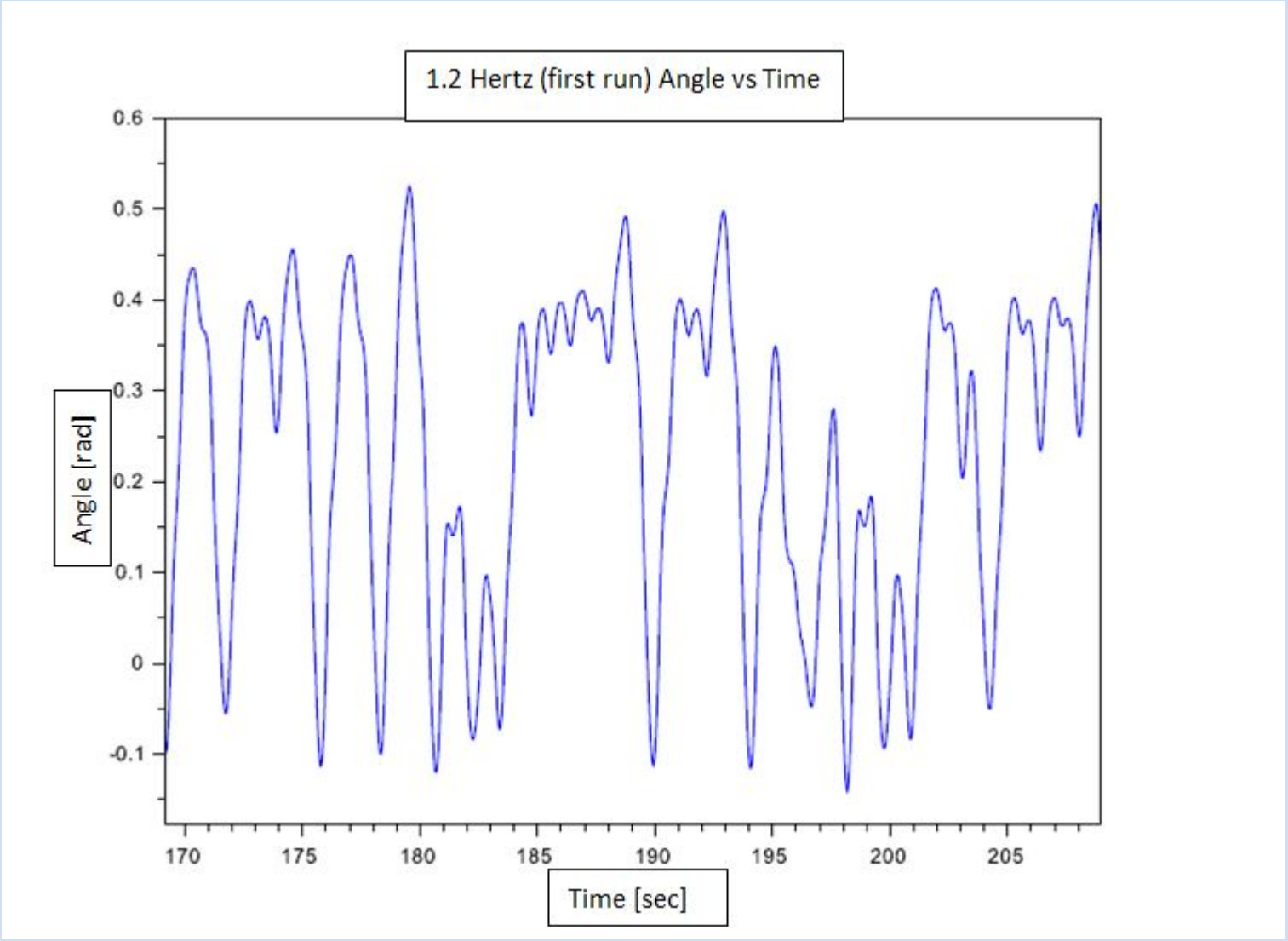
DATA COLLECTION

- Pasco Rotary Sensor
- Data Studio Software (1000 Hz – sample rate)
- Sinusoidal Function Generator to Drive the Helmholtz Coils (1 – 2 Hz)
- Support Springs to Vertically Balance Permanent Magnets on Flexible Rod (Inverted)
- 20 min. runs
- Data taken at 0.025 sec. intervals
- Velocity: time-centered average with 0.05s time step

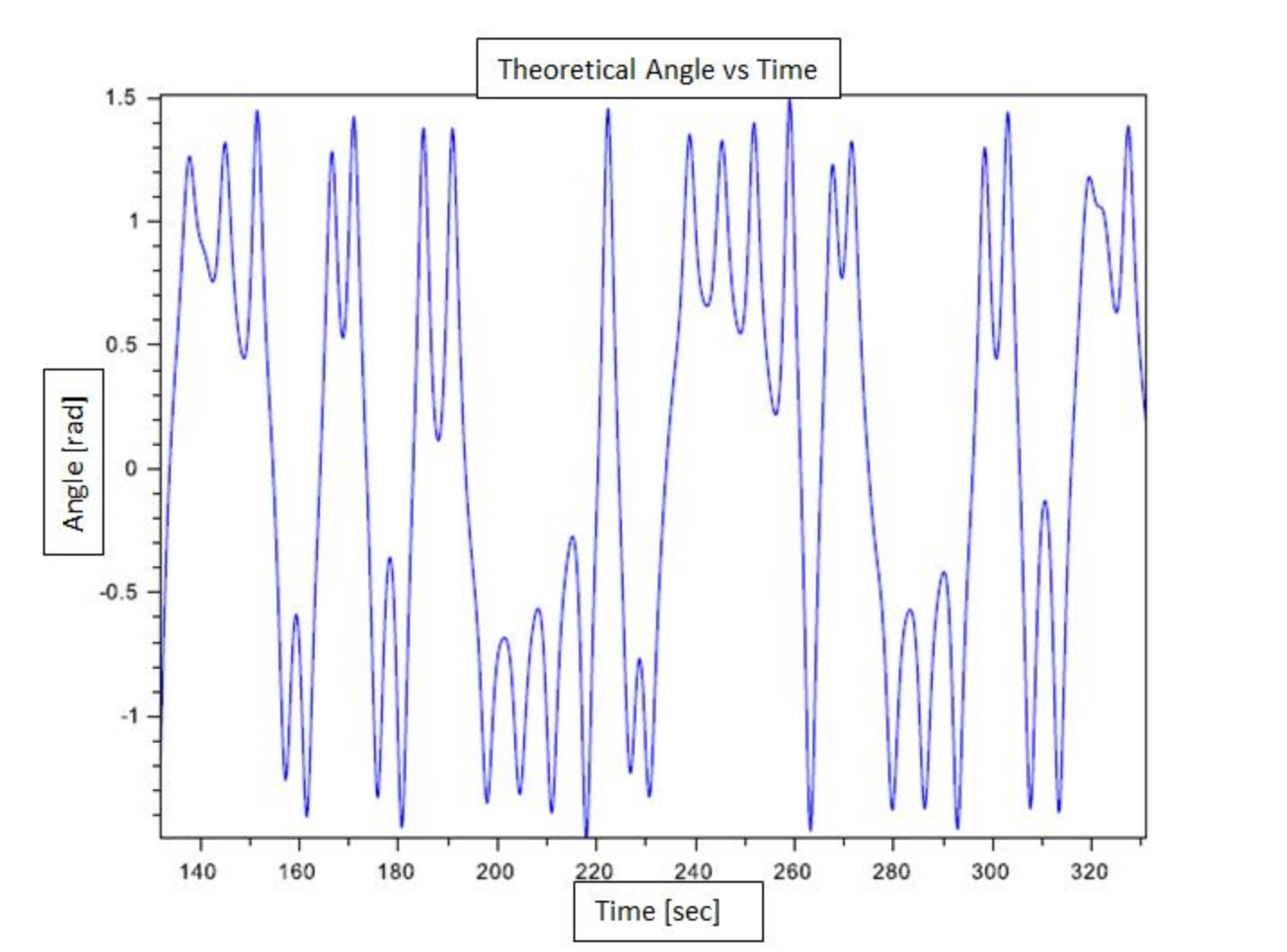
Analysis

- 1.2 million data points per typical run
  - Open source software: SciLab (developed in Europe)
  - MatLab like
  - (http://www.scilab.org/
- SciLab for simulation/analysis:
  - Scilab is free and open source software for numerical computation providing a powerful computing environment for engineering and scientific applications.
  - Scilab is released as open source under the CeCILL license (GPL compatible), and is available for download free of charge.

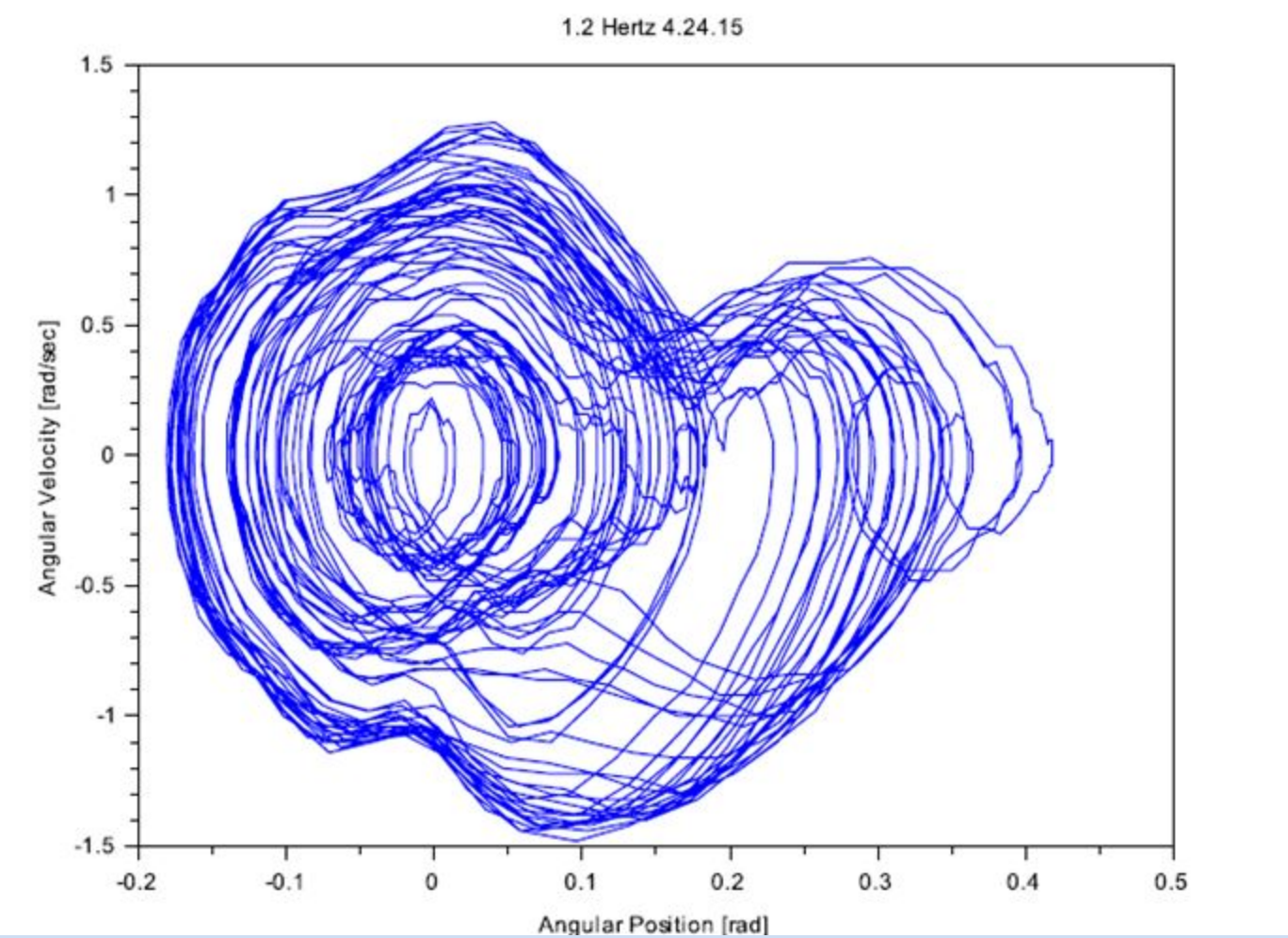
Experimental Time Series – 1.2 Hz



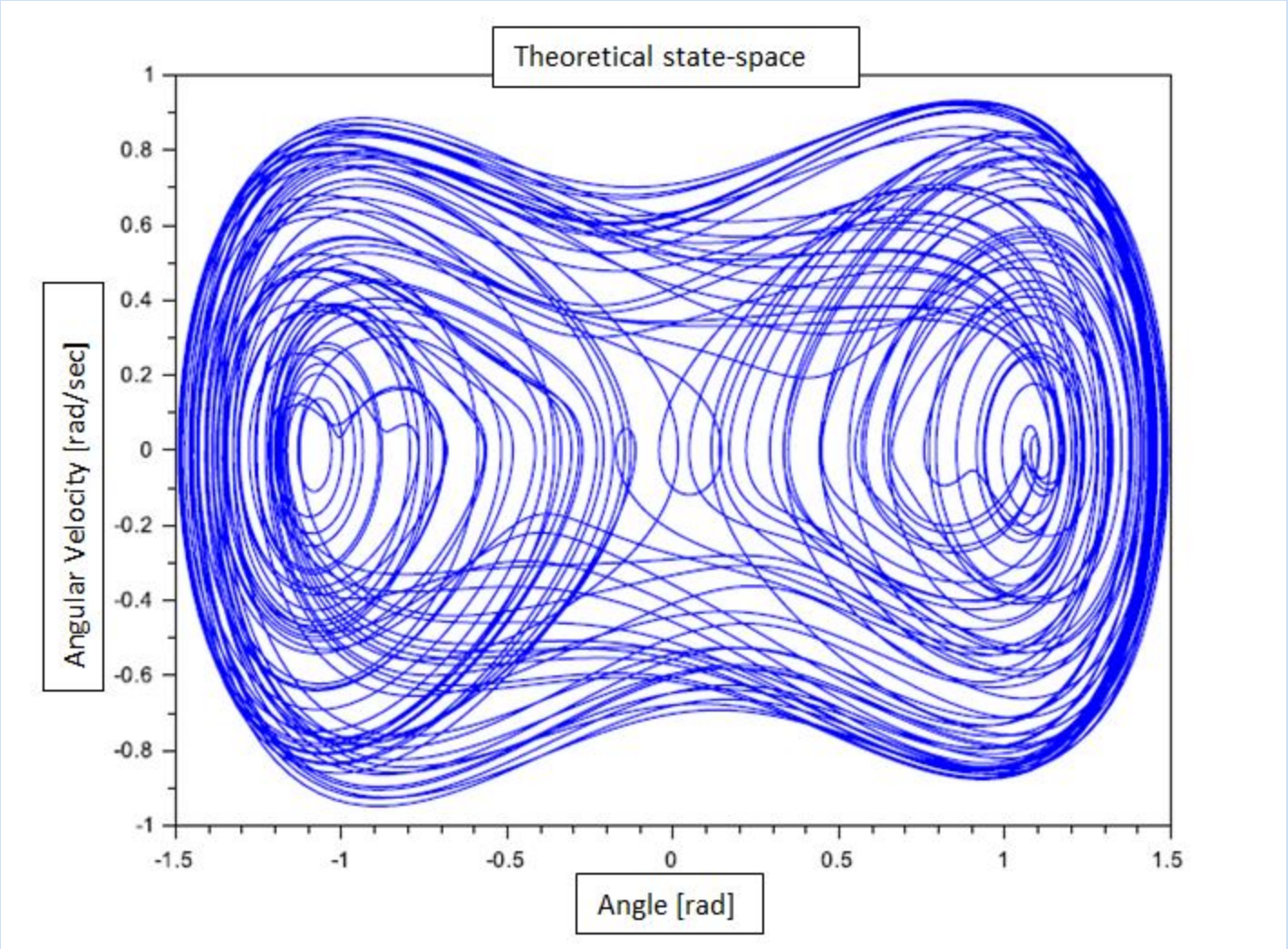
Theoretical Time Series



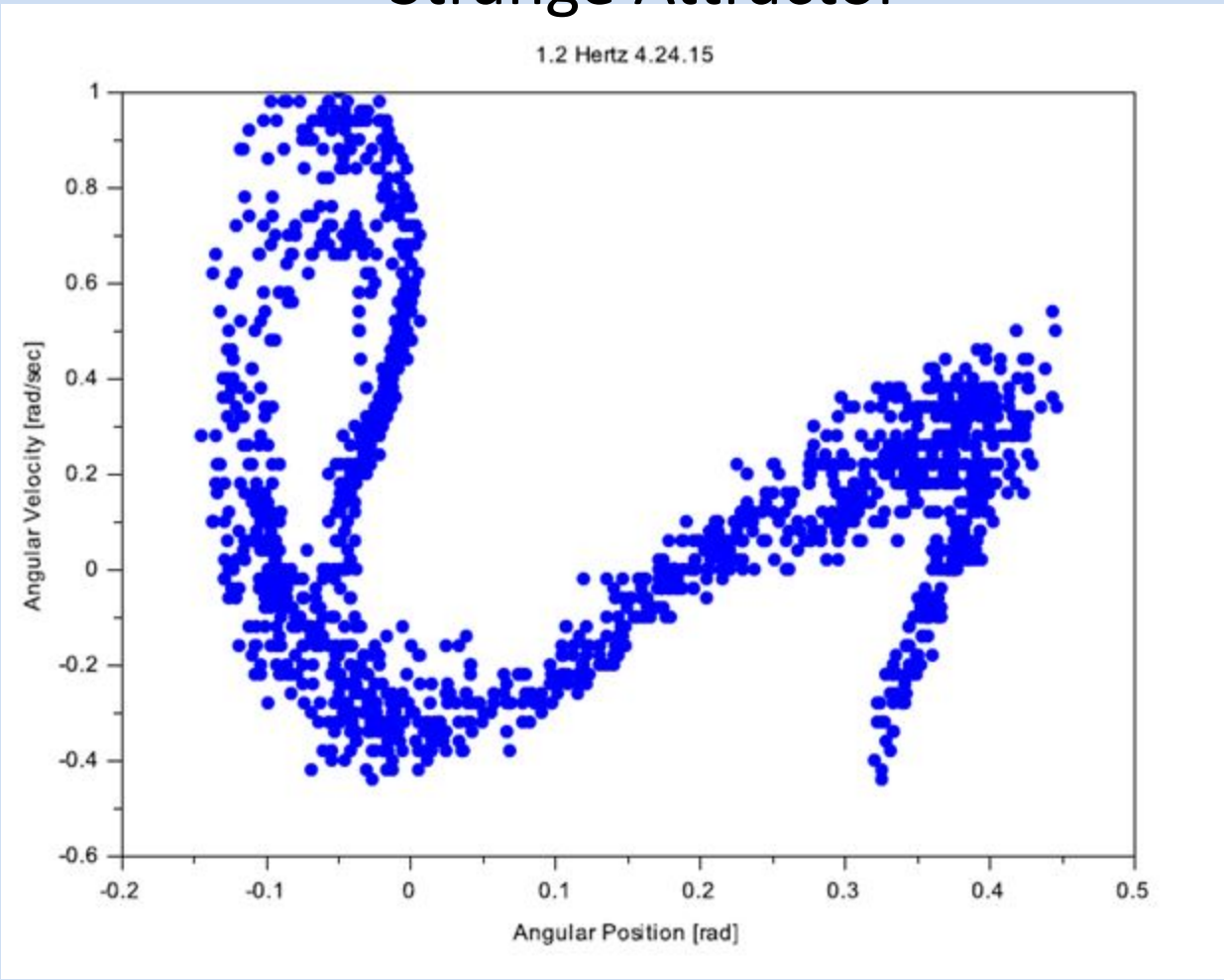
Experimental Phase Plot – 1.2 Hz



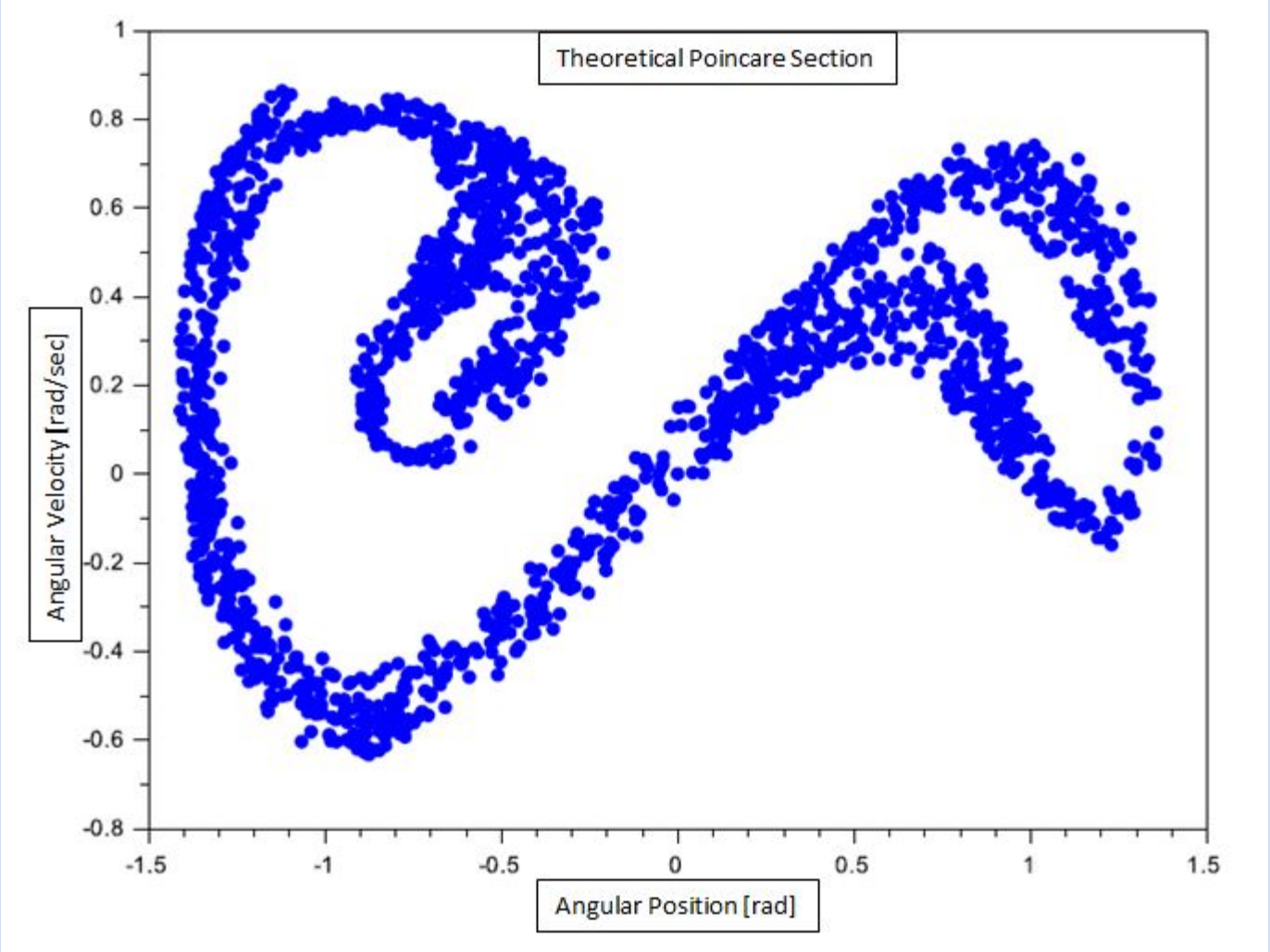
Theoretical Phase Plot



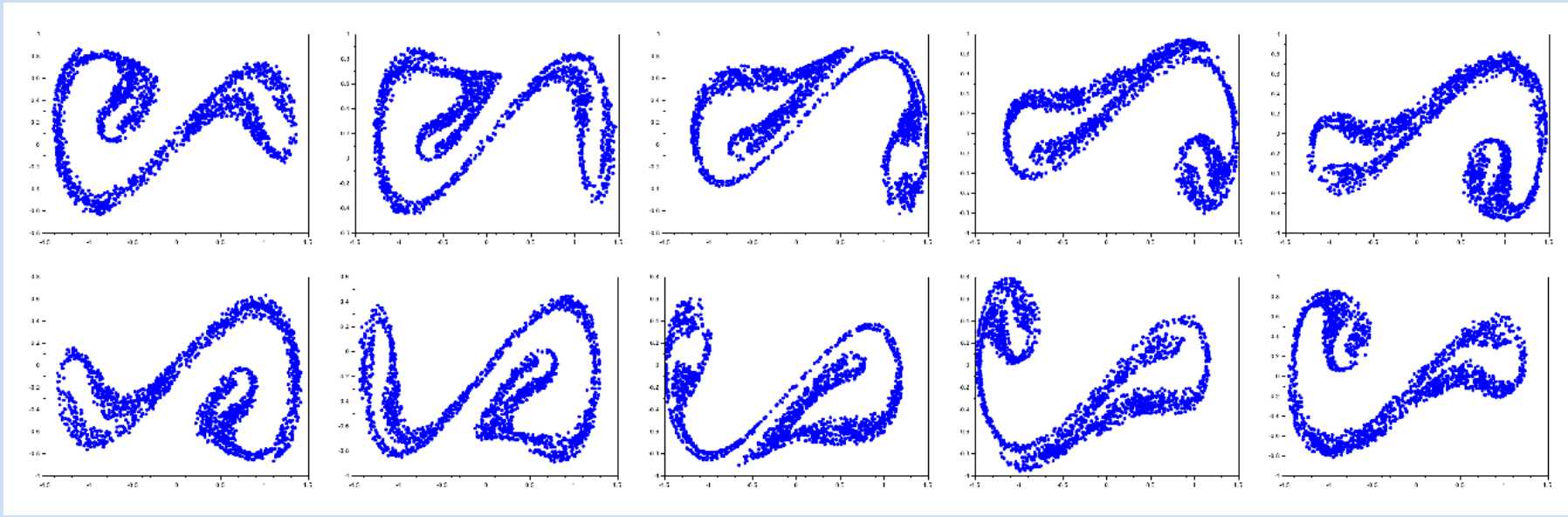
Experimental Poincare Section – 1.2 Hz Strange Attractor



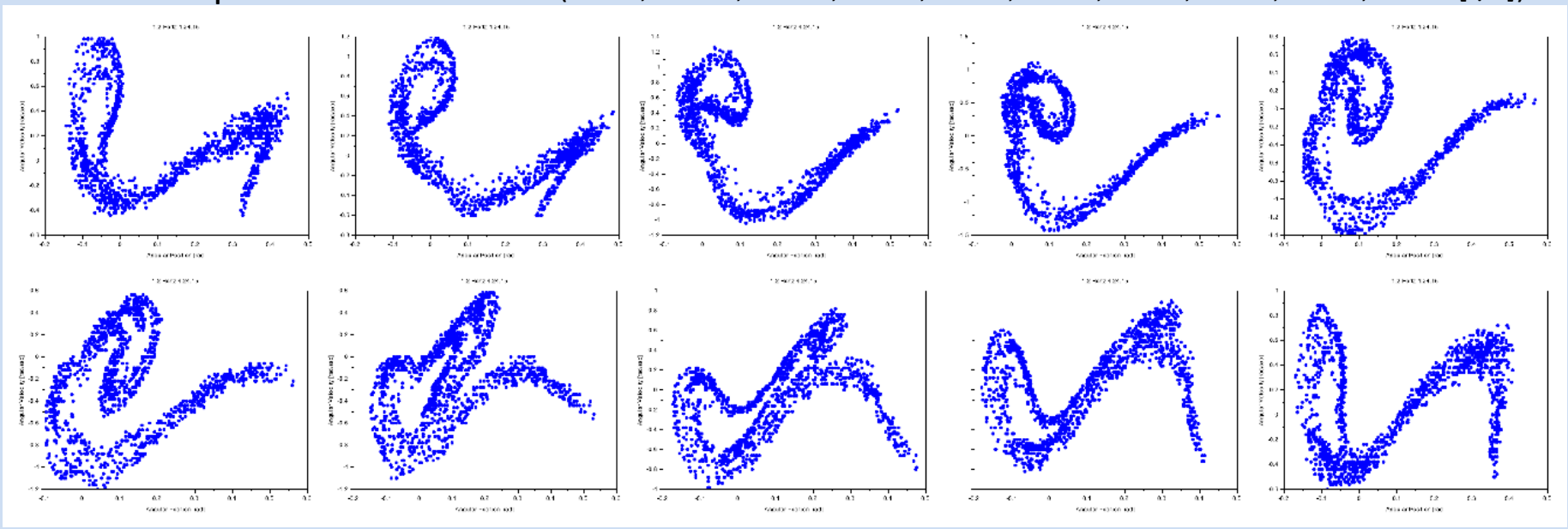
Theoretical Poincare Section
Strange Attractor



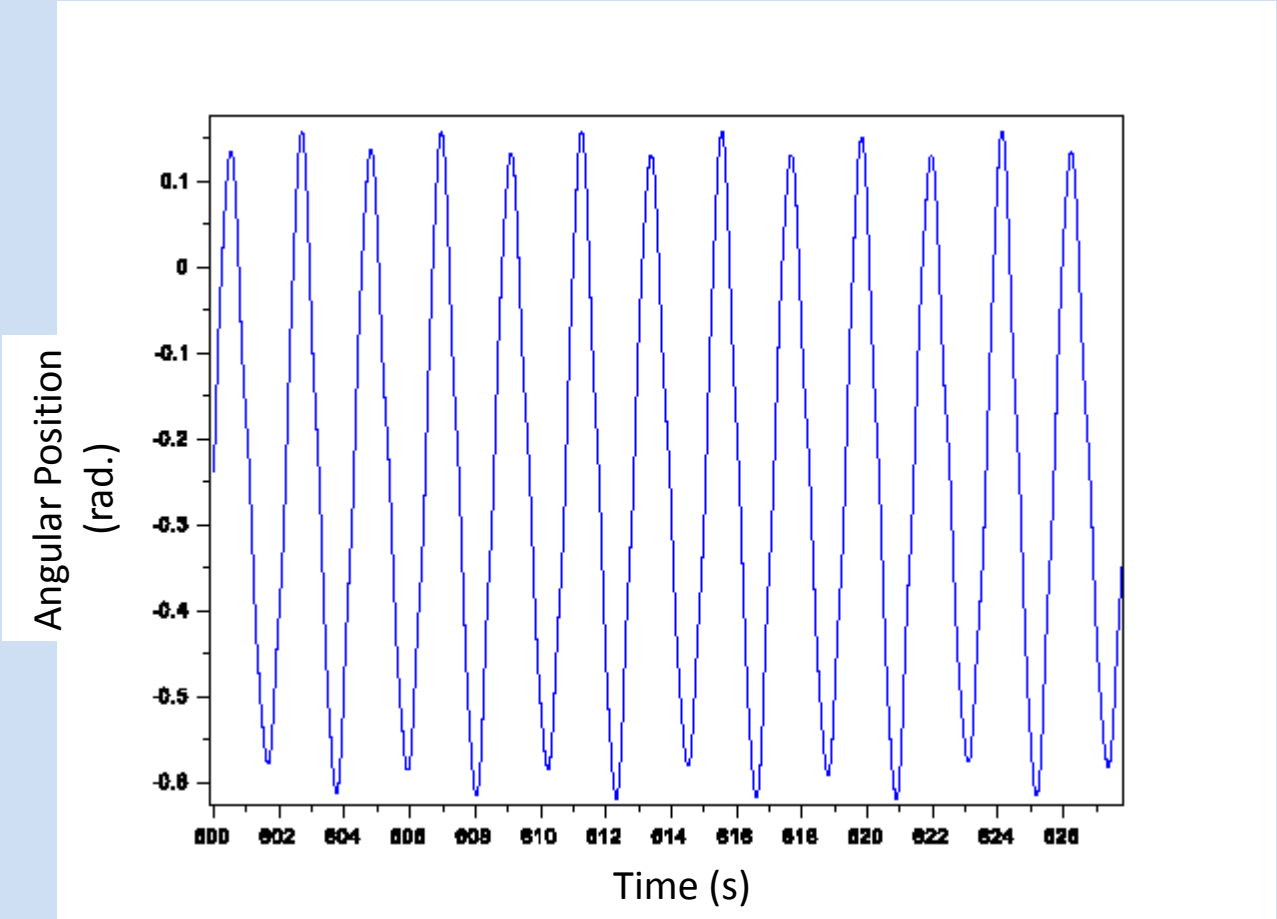
Poincare Theoretical



Poincare Experimental – 1.2 Hz (0.000, 0.083, 0.167, 0.250, 0.333, 0.417, 0.500, 0.583, 0.667, 0.750 [t/T])



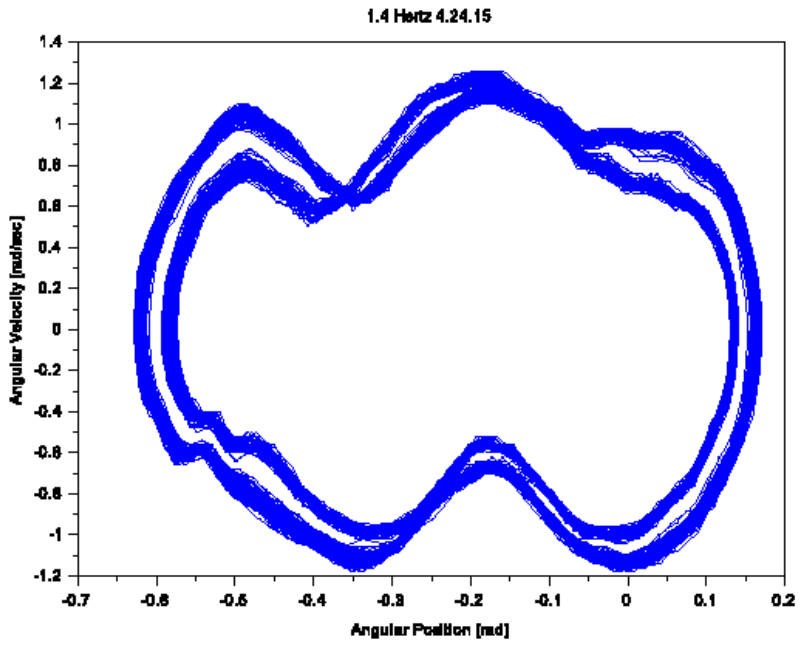
Other Behavior
(Time Series: 1.4 Hz)



Other Behavior

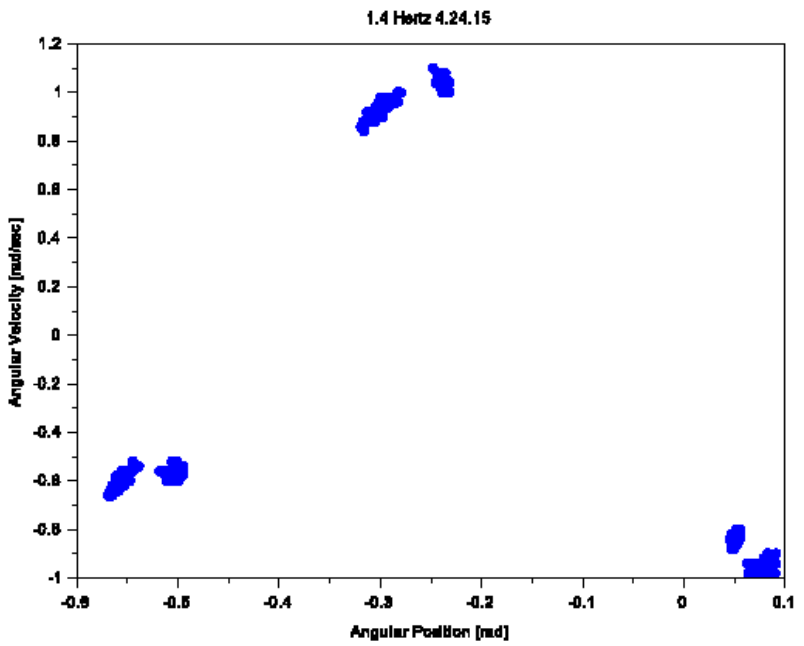
Phase Plot – 1.4 Hz

- Not Chaotic



Poincare Section – 1.4 Hz

- Period 6



Discussion

- Novel Duffing's oscillator apparatus
- Simple mechanism shows chaos
- Rich experiment with reasonable results
- Apparatus is CHEAP and easy to build from parts in most undergraduate laboratories
- Can be coupled with simulation
- And – did I mention – it was really cheap!

References

- J.E. Berger and G. Nunes Jr., “A mechanical Duffing oscillator for the undergraduate laboratory.” Am. J. Phys. **65**,841, (1997).
- H.Meissner and G. Schmidt, "A simple experiment for studying the transition from order to chaos," Am. J. Phys. **54**, 9(1986).
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- A. Cromer, “Stable solutions using the Euler approximation,” Am. J. Phys. **49**, 455 (1981).
- R. Krantz, N. Hoen, J. Zimmerman, R. Tagg, and J. Carlson, Nonlinear Dynamics on the Cheap in the Junior Laboratory, 2015 BFY Proceedings [College Park, MD, July 22-24, 2015], edited by M. Eblen-Zayas, E. Behringer, and J. Kozminski, doi:10.1119/bfy.2015.pr.012.